Dérivées des fonctions : règles suivie d'exercices

1. Dérivées simples

$$
\begin{aligned}
y = a &\Rightarrow y' = 0 \\
y = x &\Rightarrow y' = 1 \\
y = ax + b &\Rightarrow y' = a \\
y = x^n &\Rightarrow y' = n\, x^{n-1} \\
y = x^2 &\Rightarrow y' = 2x \\
y = x^5 &\Rightarrow y' = 5x^4 \\
y = \sqrt{x} = x^{\frac{1}{2}} &\Rightarrow y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{(2\sqrt{x})} \\
y = \frac{1}{x} = x^{-1} &\Rightarrow y' = -x^{-2} = \frac{-1}{x^2} \\
y = \frac{1}{x^2} = x^{-2} &\Rightarrow y' = -2x^{-3} = \frac{-2}{x^3}
\end{aligned}
$$

2. Dérivées composées

$$
\begin{aligned}
y = u + v &\Rightarrow y' = u' + v' \\
y = uv &\Rightarrow y' = u'v + uv' \\
y = \frac{u}{v} &\Rightarrow y' = \frac{u'v - uv'}{v^2} \\
y = f(u(x)) &\Rightarrow y' = f'(u) \times u' \\
y = x^2 - 1 &\Rightarrow y' = 2x \\
y = (ax + b)^n &\Rightarrow u = ax + b \Rightarrow y' = na(ax + b)^{n-1}
\end{aligned}
$$

3. Dérivées de puissances

$$
\begin{aligned}
y = \sqrt{x} = x^{\frac{1}{2}} &\Rightarrow y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \\
y = \frac{1}{x} = x^{-1} &\Rightarrow y' = -x^{-2} = -\frac{1}{x^2} \\
y = \sqrt{ax + b} &\Rightarrow u = ax + b \Rightarrow y' = \frac{a}{2\sqrt{ax + b}} \\
y = \frac{1}{ax + b} &\Rightarrow u = ax + b \Rightarrow y' = \frac{-a}{(ax + b)^2}
\end{aligned}
$$

4. Dérivées de logarithme et exponentielle

$$
\begin{aligned}
y = \ln(x) &\Rightarrow y' = \frac{1}{x} \\
y = e^x &\Rightarrow y' = e^x \\
y = \ln(x^2 - 1) &\Rightarrow u = x^2 - 1 \Rightarrow y' = \frac{2x}{x^2 - 1} \\
y = e^{x^2 - 1} &\Rightarrow u = x^2 - 1 \Rightarrow y' = 2xe^{x^2 - 1}
\end{aligned}
$$

5. EXERCICES

$$y = 2\ \ln(x) + x^2 - 1 \quad \Rightarrow \quad y' = \frac{2}{x} + 2x$$

$$y = \frac{5 - \ln(x)}{x} \quad \Rightarrow \quad u = 5 - \ln(x)\ \ v = x \Rightarrow y' = \frac{\ln(x) - 6}{x^2}$$